Unlocking the Standard Model. III. 2 Generations of quarks : calculating the Cabibbo angle

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UNLOCKING THE STANDARD MODEL

III . 2 GENERATIONS OF QUARKS : CALCULATING THE CABIBBO ANGLE

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Abstract: Maximally extending the Higgs sector of the Glashow-Salam-Weinberg model by including all scalar and pseudoscalar $J = 0$ states expected for 2 generations of quarks, I demonstrate that the Cabibbo angle is given by $\tan^2\theta_c = \dfrac{\frac{1}{m_K^2} - \frac{1}{m_D^2}}{\frac{1}{m_\pi^2} - \frac{1}{m_{D_s}^2}} \approx \dfrac{m_\pi^2}{m_K^2}\left(1 - \dfrac{m_K^2}{m_D^2} + \dfrac{m_\pi^2}{m_{D_s}^2}\right)$.

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1 Introduction

In [1] and [2], I proposed to minimally extend the Glashow-Salam-Weinberg (GSW) model [3] by maximally enlarging its Higgs sector, including in there all $J = 0$ scalar (and pseudoscalar) states that can be expected for a given number N of generation of quarks. The $8N^2$ such states, transforming like $\bar q_i q_j$ or $\bar q_i \gamma^5 q_j$ composite operators and suitably normalized can be divided into $2N^2$ quadruplets which are all in one-to-one correspondence with the complex Higgs doublet of the GSW model [4] [1]. The works [1] and [2] were dedicated to the restrictive case of 1 generation. Here I focus on the 2-generations case, but only present the calculation of the Cabibbo angle, leaving a more detailed exposition to a longer work [5].

2 Laws of transformation and isomorphism

2.1 The complex Higgs doublet of the Glashow-Salam-Weinberg model

If, instead of the customary form $H = \begin{pmatrix} \chi^1 + i\chi^2 \\ \chi^0 - ik^3 \end{pmatrix}$ involving the 4 reals fields $\chi^0, \chi^1, \chi^2, k^3 = -i\chi^3$, the complex scalar doublet H of the GSW model is written

$$H = \begin{pmatrix} h^1 - ih^2 \\ -(h^0 + h^3) \end{pmatrix}, \tag{1}$$

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[1] Some normalization factors are erroneous in [4] but have been corrected here.

the laws of transformation of its $h^0, h^j, j = 1,2,3$ components by the group $SU(2)_L$ with generators $T^i_L,\ i = 1,2,3$ write [2]

$$\boxed{\begin{array}{rcl} T^i_L \,.\, h^j &=& -\frac{1}{2}\left(i\,\epsilon_{ijk} h^k + \delta_{ij}\, h^0\right) \\ T^i_L \,.\, h^0 &=& -\frac{1}{2}\, h^i \end{array}} \tag{2}$$

Acting in the space of quark flavors (u, c, d, s) with dimension $2N = 4$, the three $SU(2)$ generators can be represented by

$$T^3 = \frac{1}{2}\left(\begin{array}{c|c} \mathbb{I} & \\ \hline & -\mathbb{I} \end{array}\right), \quad T^+ = T^1 + iT^2 = \left(\begin{array}{c|c} & \mathbb{I} \\ \hline & \end{array}\right), \quad T^- = T^1 - iT^2 = \left(\begin{array}{c|c} & \\ \hline \mathbb{I} & \end{array}\right), \tag{3}$$

where $\mathbb{I}$ is the $N \times N = 2 \times 2$ identity matrix. So doing, we realize an embedding of $SU(2)_L$ and/or $SU(2)_R$ into the chiral group $U(2N)_L \times U(2N)_R$.

2.2 Composite Higgs doublets

We now act with this chiral group on composite operators of the form $\bar\psi \mathbb{M} \psi$ and $\bar\psi\gamma^5\mathbb{M}\psi$, where ψ is the $2N$-vector of flavor quarks $\psi = (u, c, d, s)^T$ and $\mathbb{M}$ is any $2N \times 2N (= 4 \times 4)$ matrix.

$$\begin{aligned} (\mathcal{U}_L \times \mathcal{U}_R)\,.\,\bar\psi \frac{1+\gamma_5}{2}\mathbb{M}\psi &= \bar\psi\, \mathcal{U}_L^{-1}\, \mathbb{M}\, \mathcal{U}_R\, \frac{1+\gamma_5}{2}\psi, \\ (\mathcal{U}_L \times \mathcal{U}_R)\,.\,\bar\psi \frac{1-\gamma_5}{2}\mathbb{M}\psi &= \bar\psi\, \mathcal{U}_R^{-1}\, \mathbb{M}\, \mathcal{U}_L\, \frac{1-\gamma_5}{2}\psi. \end{aligned} \tag{4}$$

Writing left and right transformations of the group as

$$\mathcal{U}_{L,R} = e^{-i\alpha_i T^i_{L,R}}, \quad i = 1,2,3 \tag{5}$$

eq. (4) entails ($[\ ,\ ]$ and $\{\ ,\ \}$ stand respectively for the commutator and anticommutator)

$$\begin{aligned} T^j_L \,.\, \bar\psi\mathbb{M}\psi &= -\frac{1}{2}\left(\bar\psi\,[T^j,\mathbb{M}]\,\psi + \bar\psi\,\{T^j,\mathbb{M}\}\,\gamma_5\psi\right), \\ T^j_L \,.\, \bar\psi\mathbb{M}\gamma_5\psi &= -\frac{1}{2}\left(\bar\psi\,[T^j,\mathbb{M}]\,\gamma_5\psi + \bar\psi\,\{T^j,\mathbb{M}\}\,\psi\right), \\ T^j_R \,.\, \bar\psi\mathbb{M}\psi &= -\frac{1}{2}\left(\bar\psi\,[T^j,\mathbb{M}]\,\psi - \bar\psi\,\{T^j,\mathbb{M}\}\,\gamma_5\psi\right), \\ T^j_R \,.\, \bar\psi\mathbb{M}\gamma_5\psi &= -\frac{1}{2}\left(\bar\psi\,[T^j,\mathbb{M}]\,\gamma_5\psi - \bar\psi\,\{T^j,\mathbb{M}\}\,\psi\right). \end{aligned} \tag{6}$$

Let us consider the following set of $2N^2 = 8$ quadruplets ($\mathbb{M}^\pm = \mathbb{M}^{1\pm i2}$)

$$\bar\psi\Big(\mathbb{M}^0, \gamma^5\mathbb{M}^3, \gamma^5\mathbb{M}^+, \gamma^5\mathbb{M}^-\Big)\psi \tag{7}$$

and

$$\bar\psi\Big(\gamma^5\mathbb{M}^0, \mathbb{M}^3, \mathbb{M}^+, \mathbb{M}^-\Big)\psi, \tag{8}$$

in which

$$\mathbb{M}^0 = \left(\begin{array}{c|c} M & 0 \\ \hline 0 & M \end{array}\right), \mathbb{M}^3 = \left(\begin{array}{c|c} M & 0 \\ \hline 0 & -M \end{array}\right), \mathbb{M}^+ = 2\left(\begin{array}{c|c} 0 & M \\ \hline 0 & 0 \end{array}\right), \mathbb{M}^- = 2\left(\begin{array}{c|c} 0 & 0 \\ \hline M & 0 \end{array}\right), \tag{9}$$

[2] A transformation $\mathcal{U}_L$ of the $SU(2)_L$ group is written $\mathcal{U}_L = e^{-i\alpha_i T^i_L}, i = 1,2,3$.

M being any $N \times N = 2 \times 2$ real matrix. Denoting generically these quadruplets A and their components (a^0, a^3, a^+, a^-), their laws of transformations by $SU(2)_L$ are given by (2), in which h^0, h^i has been replaced by a^0, a^i, while they transform by $SU(2)_R$ according to

$$\begin{aligned} T^i_R \,.\, a^j &= -\tfrac{1}{2}\left(i\, \epsilon_{ijk} a^k - \delta_{ij}\, a^0\right), \\ T^i_R \,.\, a^0 &= +\tfrac{1}{2}\, a^i. \end{aligned} \tag{10}$$

We have therefore found $2N^2$ "composite" quadruplets isomorphic to the complex doublet of the GSW model. They split into N^2 of the type $(\mathfrak{s}^0, \vec{\mathfrak{p}})$ and N^2 of the type $(\mathfrak{p}^0, \vec{\mathfrak{s}})$, in which $\mathfrak{s}$ stands for "scalar" and $\mathfrak{p}$ for "pseudoscalar. These two subsets are transformed into each other by parity (the corresponding generator being $\mathbb{I}_L$ or $\mathbb{I}_R$). Their $8N^2$ components span the whole set of scalar and pseudoscalar $J = 0$ composite states that can be "built" with $2N$ quarks. In this sense, this extension represents the maximal possible extension of the Higgs sector of the GSW model.

2.3 Normalization

All composite operators that have been defined above having dimension $[mass]^3$, the quadruplets need to be suitably normalized. To this purpose we introduce $2 \times 2N^2$ parameters corresponding to the vacuum expectations values (VEV's) of, respectively:

* the scalar neutral composite operator of dimension $[mass]^3$ occurring in each quadruplet, which can only be $\mathfrak{s}^0$ or $\mathfrak{s}^3$; this VEV we call μ^3 in the first case and $\hat{\mu}^3$ in the second case, with an index that labels the quadruplet under concern;

* the corresponding scalar "Higgs" field with dimension $[mass]$; we call it $\frac{v}{\sqrt{2}}$ for an $\mathfrak{s}^0$ and $\frac{\hat{v}}{\sqrt{2}}$ for an $\mathfrak{s}^3$, with an index X, H, Ω or Ξ labeling the quadruplet under concern.

We thus consider hereafter the $N^2 = 4$ following $(\mathfrak{s}^0, \vec{\mathfrak{p}})$ quadruplets

$$\begin{aligned} X = \frac{v_X}{\sqrt{2}\,\mu_X^3}\frac{1}{\sqrt{2}}\,\bar{\psi}\left(\left(\begin{array}{cc|cc} 1 & & & \\ & 0 & & \\ \hline & & 1 & \\ & & & 0 \end{array}\right), \gamma^5\left(\begin{array}{cc|cc} 1 & & & \\ & 0 & & \\ \hline & & -1 & \\ & & & 0 \end{array}\right), 2\gamma^5\left(\begin{array}{cc|cc} & & 1 & \\ & & & 0 \\ \hline & & & \\ & & & \end{array}\right), 2\gamma^5\left(\begin{array}{cc|cc} & & & \\ & & & \\ \hline 1 & & & \\ & 0 & & \end{array}\right)\right)\psi \\ = (X^0, X^3, X^+, X^-), \quad with \quad \mu_X^3 = \frac{<\bar{u}u + \bar{d}d>}{\sqrt{2}}, \end{aligned} \tag{11}$$

$$\begin{aligned} H = \frac{v_H}{\sqrt{2}\,\mu_H^3}\frac{1}{\sqrt{2}}\,\bar{\psi}\left(\left(\begin{array}{cc|cc} 0 & & & \\ & 1 & & \\ \hline & & 0 & \\ & & & 1 \end{array}\right), \gamma^5\left(\begin{array}{cc|cc} 0 & & & \\ & 1 & & \\ \hline & & 0 & \\ & & & -1 \end{array}\right), 2\gamma^5\left(\begin{array}{cc|cc} & & 0 & \\ & & & 1 \\ \hline & & & \\ & & & \end{array}\right), 2\gamma^5\left(\begin{array}{cc|cc} & & & \\ & & & \\ \hline 0 & & & \\ & 1 & & \end{array}\right)\right)\psi \\ = (H^0, H^3, H^+, H^-), \quad with \quad \mu_H^3 = \frac{<\bar{c}c + \bar{s}s>}{\sqrt{2}}, \end{aligned} \tag{12}$$

$$
\Omega = \frac{v_\Omega}{\sqrt{2}\,\mu_\Omega^3}\frac{1}{2}\,\bar{\psi}\left(\left(\begin{array}{cc|cc} & 1 & & \\ 1 & & & \\ \hline & & & 1 \\ & & 1 & \end{array}\right), \gamma^5\left(\begin{array}{cc|cc} & 1 & & \\ 1 & & & \\ \hline & & & -1 \\ & & -1 & \end{array}\right), 2\gamma^5\left(\begin{array}{cc|cc} & & & 1 \\ & & 1 & \\ \hline & & & \\ & & & \end{array}\right), 2\gamma^5\left(\begin{array}{cc|cc} & & & \\ & & & \\ \hline & 1 & & \\ 1 & & & \end{array}\right)\right)\psi
$$

$$
= (\Omega^0, \Omega^3, \Omega^+, \Omega^-), \quad with \quad \mu_\Omega^3 = \frac{< \bar{u}c + \bar{c}u + \bar{d}s + \bar{s}d >}{2}, \tag{13}
$$

$$
\Xi = \frac{v_\Xi}{\sqrt{2}\,\mu_\Xi^3}\frac{1}{2}\,\bar{\psi}\left(\left(\begin{array}{cc|cc} & 1 & & \\ -1 & & & \\ \hline & & & 1 \\ & & -1 & \end{array}\right), \gamma^5\left(\begin{array}{cc|cc} & 1 & & \\ -1 & & & \\ \hline & & & -1 \\ & & 1 & \end{array}\right), 2\gamma^5\left(\begin{array}{cc|cc} & & & 1 \\ & & -1 & \\ \hline & & & \\ & & & \end{array}\right), 2\gamma^5\left(\begin{array}{cc|cc} & & & \\ & & & \\ \hline & 1 & & \\ -1 & & & \end{array}\right)\right)\psi
$$

$$
= (\Xi^0, \Xi^3, \Xi^+, \Xi^-), \quad with \quad \mu_\Omega^3 = \frac{< \bar{u}c - \bar{c}u + \bar{d}s - \bar{s}d >}{2}, \tag{14}
$$

and their N^2 parity transformed $(\mathfrak{p}^0, \vec{\mathfrak{s}})$ quadruplets that we call $\hat{X}, \hat{H}, \hat{\Omega}, \hat{\Xi}$. The latter are associated with the VEV's $\hat{v}_X, \hat{v}_H, \hat{v}_\Omega, \hat{v}_\Xi$, and

$$
\hat{\mu}_X^3 = \frac{< \bar{u}u - \bar{d}d >}{\sqrt{2}}, \hat{\mu}_H^3 = \frac{< \bar{c}c - \bar{s}s >}{\sqrt{2}}, \hat{\mu}_\Omega^3 = \frac{< \bar{u}c + \bar{c}u - \bar{d}s - \bar{s}d >}{2}, \hat{\mu}_\Xi^3 = \frac{< \bar{u}c - \bar{c}u - \bar{d}s + \bar{s}d >}{2}. \tag{15}
$$

We suppose that the VEV's of pseudoscalar neutral composite operators vanish, which is certainly true at the classical level (they may receive non-vanishing quantum corrections in a parity violating theory like this one, but this is beyond the scope of this work).

This makes accordingly $2 \times 2N^2$ parameters to determine, the $2N^2$ VEV's $v, \hat{v}$ of the $\mathfrak{s}^0, \mathfrak{s}^3$'s and the $2N^2$ VEV's $\mu^3, \hat{\mu}^3$ of the neutral scalar composite operators $< \bar{q}_i q_j >$.

3 The Yukawa and kinetic Lagrangians

3.1 Overview

Yukawa couplings, originally devised to trigger fermion mass terms, are built so as to be invariant by the (electro-)weak group. They are not invariant by the chiral group $U(2N)_L \times U(2N)_R$, which also makes them suitable to trigger, through low energy theorems, the masses of $J = 0$ scalar and pseudoscalar mesons. The scalar potential is chosen to be $U(2N)_L \times U(2N)_R$ chirally invariant such that all these states would be true Goldstones in the absence of Yukawa couplings and only get "soft" masses in their presence, by the effect of chiral symmetry breaking (since the weak group is a subgroup of the chiral group, weak and chiral breaking are of course entangled). The only exceptions are the 3 Goldstones of the spontaneously broken $SU(2)_L$, which should remain exactly massless and become the longitudinal components of the 3 massive W's. The scalar spectrum of the theory is therefore composed of $8N^2 - 3$ pseudo-Goldstones bosons. Some are scalars, including the "Higgs" boson and its avatars, the other are pseudoscalar mesons, which should fit those observed experimentally. The latter should in particular reproduce well known symmetry patterns which, up to a good precision, fits them into representations of a "rotated" flavor group (we call it rotated because these bound states are made with mass eigenstates and not flavor eigenstates). As far as scalar mesons are concerned, no particular symmetry structure should be found, as observed in their somewhat chaotic mass spectrum.

It may be opportune here to mention that "low energy" considerations, like the PCAC (Partially Conserved Axial Current hypothesis) and Gell-Mann-Oakes-Renner (GMOR) relations should be roughly trustable at mass scales below a few GeV's, which is much smaller that the weak scale. This is the case for 2 generations of quarks. However, when the top quark comes into the game, they should be taken with great care. This is one of the reasons why the realistic case of 3 generations is expected to be much more cumbersome that the one dealt with in this note.

3.2 The Yukawa Lagrangian

3.2.1 Its exact expression

Writing the most general such terms would mean coupling the two $SU(2)_L$ quark doublets $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ and $\begin{pmatrix} c_L \\ s_L \end{pmatrix}$ (and the 4 corresponding right-handed singlets) to the $2N^2 = 8$ available normalized Δ quadruplets (to generate masses for d-type quarks) and to their corresponding $2N^2$ conjugate alter-ego's $i\frac{T^2}{2}\Delta^*$ (to generate masses for the u-type quarks). This amounts to 64 couplings for 2 generations.

We drastically reduce their number down to 16 by comparison with what has been done in the case of 1 generation [1][2]. We write them as an "educated" quadratic sum over the N^2 set of pairs of quadruplets made of one Δ_i and its parity-transformed $\hat{\Delta}_i$, $i = X, H, \Omega, \Xi$

$$\mathcal{L}_{Yuk} = \sum_{i=X,H,\Omega,\Xi} -\delta_i\, \Delta_i^\dagger\, [\Delta_i] - \delta_{i\hat{i}}\, \Delta_i^\dagger\, [\hat{\Delta}_i] - \kappa_{\hat{i}i}\, \hat{\Delta}_i^\dagger\, [\Delta_i] - \hat{\delta}_i\, \hat{\Delta}_i^\dagger\, [\hat{\Delta}_i]. \tag{16}$$

In the formula (16), the Δ_i's and $\hat{\Delta}_i$'s stand for the complex $SU(2)_L$ doublets of the type (1) expressed in terms of quarks bilinears that are built from the quadruplets displayed in (11), (12), (13) and (14), and with their parity transformed. The $[\Delta]_i$'s and $[\hat{\Delta}_i]$'s are the (same) doublets but expressed in terms of bosonic fields with dimension $[mass]$

$$\begin{aligned} [X] &= \begin{pmatrix} [X^1] - i[X^2] \\ -([X^0] + [X^3]) \end{pmatrix}, \quad [H] = \begin{pmatrix} [H^1] - i[H^2] \\ -([H^0] + [H^3]) \end{pmatrix}, \\ [\Omega] &= \begin{pmatrix} [\Omega^1] - i[\Omega^2] \\ -([\Omega^0] + [\Omega^3]) \end{pmatrix}, \quad [\Xi] = \begin{pmatrix} [\Xi^1] - i[\Xi^2] \\ -([\Xi^0] + [\Xi^3]) \end{pmatrix}, \end{aligned} \tag{17}$$

and their parity transformed.

In the case of 1 generation, i reduces to a single value and one recovers the most general Yukawa couplings for (u, d) quarks given in eqs. (8) and (9) of [2], which is also the one of the GSW model. The expression (16) is its simplest generalization to 2 generations, in that it is the sum of the 4 similar "diagonal" contributions corresponding to the the 4 pairs $(\Delta_i, \hat{\Delta}_i)$, $i = X, H, \Omega, \Xi$, discarding all cross-couplings between different pairs $i \neq j$.

So written, $\mathcal{L}_{Yuk}$ couples the $2N$ quarks to all (pseudo-)scalar fields in a very specific way. Associated with the specific choice (11), (12), (13) and (14) for the quadruplets (any linear combinations would a priori also be a suitable possibility), it has the property of maximally avoiding flavor changing neutral currents (FCNC's) at the classical level. Introducing a coupling like $H^\dagger[X]$ would indeed generate at low energy a 4-fermion coupling proportional to $(\bar{u}\gamma^5 d)(\bar{s}\gamma^5 c)$ which carries unwanted $u \to c$ and $d \to s$ transitions. The case of crossed $\Omega - \Xi$

couplings is less evident, apart from the fact that it would generate classical transitions between $K^0 + \bar K^0$ and $K^0 - \bar K^0$. One can also argue that, formally, all quadruplets being equivalent, there is no reason to cross-couple some of them and not the others. We will show in this work and in the following ones that this choice leads to consistent results.

3.3 The kinetic Lagrangian for the scalar sector

It is

$$\mathcal{L}_{kin} = \sum_{i=X,H,\Omega,\Xi} D_\mu[\Delta_i]^\dagger D^\mu[\Delta_i] + D_\mu[\hat\Delta_i]^\dagger D^\mu[\hat\Delta_i], \tag{18}$$

where D_μ is the covariant derivative with respect to the (electro-)weak group.

The mass of the W's is accordingly given by

$$m_W^2 = \frac{g^2}{4} \sum_{i=X,H,\Omega,\Xi} (v_i^2 + \hat v_i^2). \tag{19}$$

3.4 Choosing the quasi-standard Higgs doublet

We have to make a choice concerning which quadruplet contains the 3 true Goldstone bosons of the broken $SU(2)_L$. If we choose a $(\mathfrak{s}, \vec{\mathfrak{p}})$ quadruplets, 2 charged and 1 neutral pseudoscalar mesons will automatically disappear from the spectrum. This is disfavored since all charged pseudoscalar mesons for 2 generations have been observed. If we choose $\hat\Omega$ or $\hat\Xi$, $\hat\Omega^0$ or $\hat\Xi^0$ is doomed to become the longitudinal $W^3_\parallel$; this is not good either since these are interpolating fields for neutral kaons and D mesons. We have accordingly to decide between $\hat X$ and $\hat H$. Since it looks better that the heaviest quark, the one that presumably enters into the composition of the quasi-standard Higgs boson, is called c rather than u, we choose $\hat H$ as the "quasi-standard" Higgs doublet for 2 generations.

4 Masses and orthogonality of charged pseudoscalar mesons. The Cabibbo angle

4.1 The rise of mixing

By the nature of the quadruplets $\Omega, \hat\Omega, \Xi, \hat\Xi$, their "self-coupling" occurring in the Yukawa Lagrangian triggers, through the VEV's $v_\Omega, \hat v_\Omega, v_\Xi, \hat v_\Xi$, non-diagonal fermionic mass terms $\bar u c, \bar c u, \bar d s, \bar s d$. It is then straightforward to get an expression for the $\tan$ of twice the mixing angles θ_u and θ_d in terms of Yukawa parameters.

This is however not our concern here and we shall only introduce the two mixing angles θ_u and θ_d and the quark mass eigenstates u_m, c_m, d_m, s_m as usual by (c_u, s_u mean respectively $\cos\theta_u$ and $\sin\theta_u$ *etc*)

$$\begin{pmatrix} u \\ c \end{pmatrix} = \begin{pmatrix} c_u & s_u \\ -s_u & c_u \end{pmatrix} \begin{pmatrix} u_m \\ c_m \end{pmatrix}, \quad \begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} c_d & s_d \\ -s_d & c_d \end{pmatrix} \begin{pmatrix} d_m \\ s_m \end{pmatrix}. \tag{20}$$

We shall then work at the mesonic level by using low energy theorems.

4.2 At low energy

The tools at our disposal are the statement that the divergences of axial currents of massive quarks are suitable interpolating fields for the corresponding mesons (PCAC) [6] and the Gell-Mann-Oakes-Renner relation [7] which evaluates 2-point functions of such divergences at low momentum [3].

They result, for example for the charged pions, into the 2 relations

$$\begin{aligned} &i(m_u + m_d)\bar{u}_m\gamma^5 d_m = \sqrt{2} f_\pi m_\pi^2 \pi^+, \\ &(m_u + m_d) < \bar{u}_m u_m + \bar{d}_m d_m >= 2 f_\pi^2 m_\pi^2, \end{aligned} \tag{21}$$

which evidently concern quark mass eigenstates.

With the help of these relations and equivalent, many entries of the composite quadruplets can be expressed in terms of known "particles", in particular charged pseudoscalar mesons $\pi^\pm, K^\pm, D^\pm, D_s^\pm$ [4]. This leads to the bosonised forms of the kinetic terms and Yukawa Lagrangian, valid at low energy for meson physics.

They are the ones that we use in the following and from which we request the two conditions:
$*$ no crossed terms between different charged pseudoscalar mesons should arise in the bosonised Yukawa Lagrangian;
$*$ the ratios of the quadratic terms in the Yukawa and kinetic Lagrangian for these states provide their $mass^2$.

We are careful to only use at this stage charged pseudoscalar mesons because they are experimentally observed not to mix. This is not the case for neutral pseudoscalars, the mixing pattern of which can be quite complex (and should be predictable in principle in our approach).

4.3 Notations

Because this short note does not aim at determining all parameters and because the solutions of the restricted set of equations that we shall consider for our purpose are mostly expressed in terms of the following ones, we shall define, for each pair of VEV's $(\frac{v}{\sqrt{2}}, \mu^3)$ or $(\frac{\hat{v}}{\sqrt{2}}, \hat{\mu}^3)$, the ratio with dimension $[mass]^2$

$$\nu_i^2 = \frac{\sqrt{2}\,\mu_i^3}{v_i}, \quad \hat{\nu}_i^2 = \frac{\sqrt{2}\,\hat{\mu}_i^3}{\hat{v}_i}, \quad i = X, H, \Omega, \Xi. \tag{22}$$

A priori $< \bar{u}c >=< \bar{c}u >$ and $< \bar{d}s >=< \bar{s}d >$ such that $\mu_\Xi^3 = 0$ and $\hat{\mu}_\Xi^3 = 0$. This does not mean however that v_Ξ or $\hat{v}_\Xi$ automatically vanishes.

We shall also use the following dimensionless parameters

$$b_i = \left(\frac{v_i}{\hat{v}_H}\right)^2, \quad \hat{b}_i = \left(\frac{\hat{v}_i}{\hat{v}_H}\right)^2, \quad i = X, H, \Omega, \Xi, \tag{23}$$

such that, by definition (in relation with our choice for the "quasi-standard" Higgs quadruplet that includes the 3 Goldstones of the spontaneously broken $SU(2)_L$ symmetry, see subsection 3.4)

$$\hat{b}_H = 1. \tag{24}$$

We shall also use the parameters

$$\frac{1}{\bar{\nu}_i^4} = \frac{1 - b_i}{\nu_i^4}. \tag{25}$$

[3] See also [8] and [9] for general reviews.

[4] For example, *if there was no mixing*, X^+ would write $-i\frac{v_X}{f_\pi}\pi^+$. When mixing occurs, it becomes a linear combination of π^+, K^+, D^+, D_s^+.

4.4 Mesons quadratics : orthogonality

4.4.1 Starting conditions

Charged pseudoscalar mesons only occur in the "non-hatted" bosonised quadruplets X, H, Ω, Ξ. The non-diagonal couplings between them in the bosonised Yukawa Lagrangian are proportional to the following expressions that should accordingly vanish (c_{u-d} stands for $\cos(\theta_u - \theta_d)$ *etc*)

$$\begin{aligned}
(\pi - K) &: \delta_X \frac{c_u c_d c_u s_d}{\nu_X^4} - \delta_H \frac{s_u s_d s_u c_d}{\nu_H^4} - \frac{\delta_\Omega}{2} \frac{s_{u+d} c_{u+d}}{\nu_\Omega^4} + \frac{\delta_\Xi}{2} \frac{s_{u-d} c_{u-d}}{\nu_\Xi^4} = 0, \\
(\pi - D) &: \delta_X \frac{c_u c_d s_u c_d}{\nu_X^4} - \delta_H \frac{s_u s_d c_u s_d}{\nu_H^4} - \frac{\delta_\Omega}{2} \frac{s_{u+d} c_{u+d}}{\nu_\Omega^4} - \frac{\delta_\Xi}{2} \frac{s_{u-d} c_{u-d}}{\nu_\Xi^4} = 0, \\
(\pi - D_s) &: \delta_X \frac{s_u s_d c_u c_d}{\nu_X^4} + \delta_H \frac{s_u s_d c_u c_d}{\nu_H^4} - \frac{\delta_\Omega}{2} \frac{s^2_{u+d}}{\nu_\Omega^4} + \frac{\delta_\Xi}{2} \frac{s^2_{u-d}}{\nu_\Xi^4} = 0, \\
(K - D) &: \delta_X \frac{c_u s_d s_u c_d}{\nu_X^4} + \delta_H \frac{s_u c_d c_u s_d}{\nu_H^4} + \frac{\delta_\Omega}{2} \frac{c^2_{u+d}}{\nu_\Omega^4} - \frac{\delta_\Xi}{2} \frac{c^2_{u-d}}{\nu_\Xi^4} = 0, \\
(K - D_s) &: \delta_X \frac{c_u s_d s_u s_d}{\nu_X^4} - \delta_H \frac{s_u c_d c_u c_d}{\nu_H^4} + \frac{\delta_\Omega}{2} \frac{s_{u+d} c_{u+d}}{\nu_\Omega^4} + \frac{\delta_\Xi}{2} \frac{s_{u-d} c_{u-d}}{\nu_\Xi^4} = 0, \\
(D - D_s) &: \delta_X \frac{s_u c_d s_u s_d}{\nu_X^4} - \delta_H \frac{c_u s_d c_u c_d}{\nu_H^4} + \frac{\delta_\Omega}{2} \frac{s_{u+d} c_{u+d}}{\nu_\Omega^4} - \frac{\delta_\Xi}{2} \frac{s_{u-d} c_{u-d}}{\nu_\Xi^4} = 0.
\end{aligned} \tag{26}$$

4.5 Basics for the scalar potential. Connecting the δ_i's.

Relations between $\delta_X, \delta_H, \delta_\Omega, \delta_\Xi$ can be obtained by minimizing the effective potential $V_{eff}(\Delta_i)$ obtained by subtracting the bosonised Yukawa Lagrangian [5]

$$\mathcal{L}^{bos}_{Yuk} = \sum_{i=X,H,\Omega,\Xi} -\delta_i\, [\Delta_i]^\dagger\, [\Delta_i] - \frac{1}{2}(\delta_{i\hat{i}} + \kappa_{\hat{i}i}) \left([\Delta_i]^\dagger\, [\hat{\Delta}_i] + [\hat{\Delta}_i]^\dagger\, [\Delta_i] \right) - \hat{\delta}_i\, [\hat{\Delta}_i]^\dagger\, [\hat{\Delta}_i] \tag{27}$$

to the scalar potential $V(\Delta_i)$ suitably chosen. To this purpose, it is most efficient to work in "flavor space", which means here using the components $\Delta_i^0, \Delta_i^3, \Delta_i^+, \Delta_i^-$ of each quadruplet Δ_i and not the meson fields like $\pi, K \ldots$

There again, the choice of V is important. The most general scalar potential for $2N^2 = 8$ Higgs multiplets has a large number of parameters. However, as we already mentioned, we choose it to be $U(2N)_L \times U(2N)_R$ chirally invariant and such that no nonphysical transition between known particles, nor any unrealistic mass splitting gets induced at the classical level. These requirements lead to an extremely simple form, like for the Yukawa Lagrangian, which is

$$V = -\frac{m_H^2}{2} \sum_i \Delta_i^\dagger \Delta_i + \frac{\lambda_H}{4} \sum_i (\Delta_i^\dagger \Delta_i)^2, \quad i = X, H, \Omega, \Xi, \hat{X}, \hat{H}, \hat{\Omega}, \hat{\Xi}. \tag{28}$$

It only involves 2 parameters, m_H^2 and λ_H. The effective potential $V_{eff} = V - \mathcal{L}_{Yuk}$ therefore involves 18 unknown parameters.

The bosonised Yukawa Lagrangian gets simplified by requesting that charged pseudoscalar and scalar mesons do not couple at the classical level. This requires

$$\delta_{i\hat{i}} + \kappa_{\hat{i}i} = 0, \quad i = X, H, \Omega, \Xi. \tag{29}$$

[5]This symmetric and hermitian form is obtained by simply rewriting all terms in the Yukawa Lagrangian (16) in terms of fields of dimension $mass$ like in [2].

Minimizing V_{eff} at the values $< X^0 >= \frac{v_X}{\sqrt{2}}, < \hat{H}^3 >= \frac{\hat{v}_H}{\sqrt{2}} \ldots$ yields then $2 \times 4 = 8$ equations of the type

$$m_H^2 = \lambda_H \frac{v_i^2}{2} + 2\delta_i, \ldots \quad m_H^2 = \lambda_H \frac{\hat{v}_i^2}{2} + 2\hat{\delta}_i, \ldots \tag{30}$$

One among them is special, the one related to the "quasi-standard" Higgs doublet $\hat{H}$. That the 3 Goldstone bosons of the broken chiral symmetry that it contains stay as the 3 true Goldstones of the spontaneously broken $SU(2)_L$ requires in particular

$$\hat{\delta}_H = 0, \tag{31}$$

which entails

$$m_H^2 = \lambda_H \frac{\hat{v}_H^2}{2}, \tag{32}$$

and thus

$$\lambda_H = \frac{4\delta_i}{\hat{v}_H^2 - v_i^2} = \frac{4\hat{\delta}_i}{\hat{v}_H^2 - \hat{v}_i^2}. \tag{33}$$

Let us define δ such that [6]

$$\lambda_H = \frac{4\delta}{\hat{v}_H^2} \Rightarrow m_H^2 = 2\delta. \tag{34}$$

Then

$$\delta_i = \delta(1 - b_i), \quad \hat{\delta} = \delta(1 - \hat{b}_i). \tag{35}$$

4.6 Solution of the equations (26)

Using the relations (35) between the δ_i and (25), $\delta \neq 0$ can be factored out and equations (26) rewrite

$$\begin{aligned}
(a) &: \frac{c_u c_d c_u s_d}{\bar{\nu}_X^4} - \frac{s_u s_d s_u c_d}{\bar{\nu}_H^4} - \frac{1}{2}\frac{s_{u+d}\, c_{u+d}}{\bar{\nu}_\Omega^4} + \frac{1}{2}\frac{s_{u-d}\, c_{u-d}}{\bar{\nu}_\Xi^4} = 0,\\
(b) &: \frac{c_u c_d s_u c_d}{\bar{\nu}_X^4} - \frac{s_u s_d c_u s_d}{\bar{\nu}_H^4} - \frac{1}{2}\frac{s_{u+d}\, c_{u+d}}{\bar{\nu}_\Omega^4} - \frac{1}{2}\frac{s_{u-d}\, c_{u-d}}{\bar{\nu}_\Xi^4} = 0,\\
(c) &: \frac{s_u s_d c_u c_d}{\bar{\nu}_X^4} + \frac{s_u s_d c_u c_d}{\bar{\nu}_H^4} - \frac{1}{2}\frac{s_{u+d}^2}{\bar{\nu}_\Omega^4} + \frac{1}{2}\frac{s_{u-d}^2}{\bar{\nu}_\Xi^4} = 0,\\
(d) &: \frac{c_u s_d s_u c_d}{\bar{\nu}_X^4} + \frac{s_u c_d c_u s_d}{\bar{\nu}_H^4} + \frac{1}{2}\frac{c_{u+d}^2}{\bar{\nu}_\Omega^4} - \frac{1}{2}\frac{c_{u-d}^2}{\bar{\nu}_\Xi^4} = 0,\\
(e) &: \frac{c_u s_d s_u s_d}{\bar{\nu}_X^4} - \frac{s_u c_d c_u c_d}{\bar{\nu}_H^4} + \frac{1}{2}\frac{s_{u+d}\, c_{u+d}}{\bar{\nu}_\Omega^4} + \frac{1}{2}\frac{s_{u-d}\, c_{u-d}}{\bar{\nu}_\Xi^4} = 0,\\
(f) &: \frac{s_u c_d s_u s_d}{\bar{\nu}_X^4} - \frac{c_u s_d c_u c_d}{\bar{\nu}_H^4} + \frac{1}{2}\frac{s_{u+d}\, c_{u+d}}{\bar{\nu}_\Omega^4} - \frac{1}{2}\frac{s_{u-d}\, c_{u-d}}{\bar{\nu}_\Xi^4} = 0,
\end{aligned} \tag{36}$$

or, equivalently, by recombining the equation

$$\begin{aligned}
(a) + (f) &: s_{2d}\left(\frac{1}{\bar{\nu}_X^4} - \frac{1}{\bar{\nu}_H^4}\right) = 0,\\
(a) - (f) &: s_{2d}c_{2u}\left(\frac{1}{\bar{\nu}_X^4} + \frac{1}{\bar{\nu}_H^4}\right) - \frac{s_{2(u+d)}}{\bar{\nu}_\Omega^4} + \frac{s_{2(u-d)}}{\bar{\nu}_\Xi^4} = 0,\\
(b) - (e) &: s_{2u}c_{2d}\left(\frac{1}{\bar{\nu}_X^4} + \frac{1}{\bar{\nu}_H^4}\right) - \frac{s_{2(u+d)}}{\bar{\nu}_\Omega^4} - \frac{s_{2(u-d)}}{\bar{\nu}_\Xi^4} = 0,\\
(b) + (e) &: s_{2u}\left(\frac{1}{\bar{\nu}_X^4} - \frac{1}{\bar{\nu}_H^4}\right) = 0,\\
(c) - (d) &: \frac{1}{\bar{\nu}_\Omega^4} - \frac{1}{\bar{\nu}_\Xi^4} = 0,\\
(c) + (d) &: s_{2u}s_{2d}\left(\frac{1}{\bar{\nu}_X^4} + \frac{1}{\bar{\nu}_H^4}\right) + \frac{c_{2(u+d)}}{\bar{\nu}_\Omega^4} - \frac{c_{2(u-d)}}{\bar{\nu}_\Xi^4} = 0.
\end{aligned} \tag{37}$$

[6]The mass scale set by δ, tightly connected with the mass of the "quasi-standard" Higgs boson, can be evaluated by looking at neutral kaons and D mesons. We do not need it here and therefore delay its presentation to [5].

The solution of (37) is

$$\frac{1}{\bar{\nu}_X^4} = \frac{1}{\bar{\nu}_H^4} = \frac{1}{\bar{\nu}_\Omega^4} = \frac{1}{\bar{\nu}_\Xi^4} \overset{(25)}{\Leftrightarrow} \frac{1-b_X}{\nu_X^4} = \frac{1-b_H}{\nu_H^4} = \frac{1-b_\Omega}{\nu_\Omega^4} = \frac{1-b_\Xi}{\nu_\Xi^4}. \tag{38}$$

4.7 Mesons quadratics : masses

From the ratios of the terms quadratic in the meson fields in the effective potential and in the kinetic terms, using (35) and (23) one gets

$$\begin{aligned}
m_{\pi^\pm}^2 &= \delta \frac{(1-b_X)\,(\frac{c_u c_d}{\nu_X^2})^2 + (1-b_H)\,(\frac{s_u s_d}{\nu_H^2})^2 + (1-b_\Omega)\,\frac{1}{2}(\frac{s_{u+d}}{\nu_\Omega^2})^2 + (1-b_\Xi)\,\frac{1}{2}(\frac{s_{u-d}}{\nu_\Xi^2})^2}{(\frac{c_u c_d}{\nu_X^2})^2 + (\frac{s_u s_d}{\nu_H^2})^2 + \frac{1}{2}(\frac{s_{u+d}}{\nu_\Omega^2})^2 + \frac{1}{2}(\frac{s_{u-d}}{\nu_\Xi^2})^2},\\
m_{K^\pm}^2 &= \delta \frac{(1-b_X)\,(\frac{c_u s_d}{\nu_X^2})^2 + (1-b_H)\,(\frac{s_u c_d}{\nu_H^2})^2 + (1-b_\Omega)\,\frac{1}{2}(\frac{c_{u+d}}{\nu_\Omega^2})^2 + (1-b_\Xi)\,\frac{1}{2}(\frac{c_{u-d}}{\nu_\Xi^2})^2}{(\frac{c_u s_d}{\nu_X^2})^2 + (\frac{s_u c_d}{\nu_H^2})^2 + \frac{1}{2}(\frac{c_{u+d}}{\nu_\Omega^2})^2 + \frac{1}{2}(\frac{c_{u-d}}{\nu_\Xi^2})^2},\\
m_{D^\pm}^2 &= \delta \frac{(1-b_X)\,(\frac{s_u c_d}{\nu_X^2})^2 + (1-b_H)\,(\frac{c_u s_d}{\nu_H^2})^2 + (1-b_\Omega)\,\frac{1}{2}(\frac{c_{u+d}}{\nu_\Omega^2})^2 + (1-b_\Xi)\,\frac{1}{2}(\frac{c_{u-d}}{\nu_\Xi^2})^2}{(\frac{s_u c_d}{\nu_X^2})^2 + (\frac{c_u s_d}{\nu_H^2})^2 + \frac{1}{2}(\frac{c_{u+d}}{\nu_\Omega^2})^2 + \frac{1}{2}(\frac{c_{u-d}}{\nu_\Xi^2})^2},\\
m_{D_s^\pm}^2 &= \delta \frac{(1-b_X)\,(\frac{s_u s_d}{\nu_X^2})^2 + (1-b_H)\,(\frac{c_u c_d}{\nu_H^2})^2 + (1-b_\Omega)\,\frac{1}{2}(\frac{s_{u+d}}{\nu_\Omega^2})^2 + (1-b_\Xi)\,\frac{1}{2}(\frac{s_{u-d}}{\nu_\Xi^2})^2}{(\frac{s_u s_d}{\nu_X^2})^2 + (\frac{c_u c_d}{\nu_H^2})^2 + \frac{1}{2}(\frac{s_{u+d}}{\nu_\Omega^2})^2 + \frac{1}{2}(\frac{s_{u-d}}{\nu_\Xi^2})^2},
\end{aligned} \tag{39}$$

which rewrites, using (38)

$$\begin{aligned}
m_{\pi^\pm}^2 &= \frac{\delta/\bar{\nu}_X^4}{(c_u c_d/\nu_X^2)^2 + (s_u s_d/\nu_H^2)^2 + \frac{1}{2}(s_{u+d}/\nu_\Omega^2)^2 + \frac{1}{2}(s_{u-d}/\nu_\Xi^2)^2},\\
m_{K^\pm}^2 &= \frac{\delta/\bar{\nu}_X^4}{(c_u s_d/\nu_X^2)^2 + (s_u c_d/\nu_H^2)^2 + \frac{1}{2}(c_{u+d}/\nu_\Omega^2)^2 + \frac{1}{2}(c_{u-d}/\nu_\Xi^2)^2},\\
m_{D^\pm}^2 &= \frac{\delta/\bar{\nu}_X^4}{(s_u c_d/\nu_X^2)^2 + (c_u s_d/\nu_H^2)^2 + \frac{1}{2}(c_{u+d}/\nu_\Omega^2)^2 + \frac{1}{2}(c_{u-d}/\nu_\Xi^2)^2},\\
m_{D_s^\pm}^2 &= \frac{\delta/\bar{\nu}_X^4}{(s_u s_d/\nu_X^2)^2 + (c_u c_d/\nu_H^2)^2 + \frac{1}{2}(s_{u+d}/\nu_\Omega^2)^2 + \frac{1}{2}(s_{u-d}/\nu_\Xi^2)^2}.
\end{aligned} \tag{40}$$

Eqs. (40) entail

$$\begin{aligned}
\delta\left(+\frac{1}{m_{\pi^\pm}^2} + \frac{1}{m_{K^\pm}^2} + \frac{1}{m_{D^\pm}^2} + \frac{1}{m_{D_s^\pm}^2}\right) &= \frac{1}{1-b_X} + \frac{1}{1-b_H} + \frac{1}{1-b_\Omega} + \frac{1}{1-b_\Xi},\\
\delta\left(+\frac{1}{m_{\pi^\pm}^2} - \frac{1}{m_{K^\pm}^2} + \frac{1}{m_{D^\pm}^2} - \frac{1}{m_{D_s^\pm}^2}\right) &= c_{2d}\left(\frac{1}{1-b_X} - \frac{1}{1-b_H}\right),\\
\delta\left(+\frac{1}{m_{\pi^\pm}^2} + \frac{1}{m_{K^\pm}^2} - \frac{1}{m_{D^\pm}^2} - \frac{1}{m_{D_s^\pm}^2}\right) &= c_{2u}\left(\frac{1}{1-b_X} - \frac{1}{1-b_H}\right),\\
\delta\left(+\frac{1}{m_{\pi^\pm}^2} - \frac{1}{m_{K^\pm}^2} - \frac{1}{m_{D^\pm}^2} + \frac{1}{m_{D_s^\pm}^2}\right) &= c_{2u}c_{2d}\left(\frac{1}{1-b_X} + \frac{1}{1-b_H}\right) - \frac{c_{2(u+d)}}{1-b_\Omega} - \frac{c_{2(u-d)}}{1-b_\Xi}.
\end{aligned} \tag{41}$$

4.8 The Cabibbo angle

From the second and third equations of (41) one gets, independently of the scale δ

$$\frac{c_{2u}-c_{2d}}{c_{2u}+c_{2d}} \equiv \tan(\theta_d+\theta_u)\tan(\theta_d-\theta_u) = \frac{\frac{1}{m_{K^\pm}^2} - \frac{1}{m_{D^\pm}^2}}{\frac{1}{m_{\pi^\pm}^2} - \frac{1}{m_{D_s^\pm}^2}}, \tag{42}$$

which vanishes either at the chiral limit $m_\pi \to 0$ or when $m_K = m_D$.

By the freedom to make an arbitrary flavor rotation on (u,c) quarks, one can align flavor and mass eigenstates in this sector and, therefore, tune $\theta_u \to 0$. θ_d becomes then the Cabibbo angle θ_c which is given by

$$\boxed{\tan^2\theta_c = \frac{\dfrac{1}{m^2_{K^\pm}} - \dfrac{1}{m^2_{D^\pm}}}{\dfrac{1}{m^2_{\pi^\pm}} - \dfrac{1}{m^2_{D_s^\pm}}} \approx \frac{m^2_{\pi^\pm}}{m^2_{K^\pm}}\left(1 - \frac{m^2_{K^\pm}}{m^2_{D^\pm}} + \frac{m^2_{\pi^\pm}}{m^2_{D_s^\pm}}\right)} \qquad q.e.d. \tag{43}$$

Numerically, it corresponds to $\theta_c \approx .27$, to be compared with the measured $\approx .23$.

5 Conclusion and prospects

With the example of the Cabibbo angle, we have shown that the extension that we propose for the GSW model allows calculations that have long been sought for [7]. This angle we determined from the sole physical data concerning the masses and orthogonality of the 4 types of charged pseudoscalar mesons $\pi^\pm, K^\pm, D^\pm$ and $D_s^\pm$, such that we had only to exploit a small part of the physical information available concerning pseudoscalar mesons.

While the Higgs sector of the GSW model has been maximally extended by including in it all $J=0$ mesons expected for a given number of generations of quarks, the Yukawa Lagrangian and the scalar potential have been reduced to very simple expressions by requirements of invariance and to avoid classical unwanted phenomena like FCNC, non-existing crossed couplings between known states and unrealistic mass differences (like, in the case of 1 generation, $\pi^+ - \pi^0$ mass difference which originates neither from $m_d \neq m_u$ nor from electromagnetic corrections). This makes this extension the simplest, minimal and most natural one, showing that these criteria may be at work once more in nature.

Obtaining a sensible expression for the Cabibbo angle suggests that this direction is worth detailed investigations. I shall present in a forthcoming work [5], still for 2 generations, the values of all VEV's, the masses of all Higgs bosons and their couplings to gauge bosons and to fermions.

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